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Oscillating Two-Dimensional Hypersonic Airfoils at Small Angles of Attack

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Pitching oscillations of two-dimensional pointed-nose thin airfoils with small surface curvature are considered in this paper. The analysis relies on recently developed theories for steady and unsteady hypersonic flows past such airfoils. For small surface curvature τ and small reduced frequency k, a double series in τ and k is assumed here and shown to lead to very simple systems of linear equations having first- or second-degree polynomial solutions. Thus, simple closed-form formulas for the unsteady surface pressure and the stability derivatives of any curved nonsymmetric airfoil (with different upper and lower surface shapes), pitching at a small angle of attack, are obtained. Results for symmetric wedges at zero incidence are compared with other available analytical and experimental calculations and the agreement is found to be generally good.

I. Introduction

RECENTLY,[1-4] new hypersonic approximations were presented for steady and unsteady flows past pointed-nose thin airfoils at small incidence. Through careful investigation of the order of magnitude of the various terms of the steady and unsteady inviscid flow equations and boundary conditions, it was possible to reduce the steady flow equations to a form somewhat similar to the hypersonic small disturbance theory (HSDT), but with the additional advantage of obtaining a first integral whereby the density was eliminated from the formulation, thus reducing by one the number of unknown functions [see Ref. 2, Eqs. (6)].

The unsteady flow approximate equations [Ref. 3, Eqs. (14-16)] are linear and were used[3] to derive a zeroth-order unsteady Newtonian theory showing the effect of surface curvature but not showing the effect of γ and M_∞, where γ is the ratio of the specific heats of the gas and M_∞ is the freestream Mach number. In Ref. 4, the unsteady theory was further restricted to moderate supersonic Mach numbers [Ref. 4, Eqs. (17-28)]. Simple closed-form formulas for the stability derivatives were found and used to present results for the stability derivatives and the neutral stability boundary.

Unsteady supersonic flow has received great attention in the literature over the past decades. Besides the potential flow theory[5] and the HSDT,[6] the perturbation approach has been extensively used to study unsteady flow past oscillating wedges (and delta wings and circular cones). Thus, there exists the theories of Appleton,[7] McIntosh,[8] Orlik-Rückemann,[9] and Hui,[10-12] among others.[13,14] The effect of viscosity on the stability of wedges was also considered by Orlik-Rückemann[15] and by Hui and East.[16]

Recently, Hui and Tobak[17] and Hui[18] presented interesting theories extending the Newton-Busemann pressure law to unsteady flow past pitching oscillating two-dimensional airfoils.

The case of pitching oscillations at moderate or large angles of attack was also considered by Hui and Tobak[19] for an aircraft and Hui et al.[20] for three-dimensional flat wings, whereas the case of a two-dimensional airfoil at such angles of attack was considered by Hemdan.[21]

The purpose of this paper is to find solutions to the unsteady flow theory given in Ref. 3 by covering the range of M_∞ extending from moderate supersonic Mach numbers to Newtonian flow, thus including the results of Refs. 3 and 4 as special cases. In Sec. II, we give a brief summary of the unsteady hypersonic theory and derive a first integral for it. In Sec. III, the unsteady equations are perturbed for small τ and small k by assuming a double series expansion in the two parameters. In Sec. IV, simple closed-form formulas for the unsteady surface pressure are found and used in Sec. V to find the stability derivatives of any nonsymmetric curved airfoil. In Sec. VI, we present results for the stability derivatives and compare our results for wedges pitching at zero incidence with other available results. In Sec. VII, we give some concluding remarks.

II. First Integral for Unsteady Flow

Consider a two-dimensional pointed nose thin airfoil of length l to be making small amplitude pitching oscillations about an axis at a distance $\bar{h}$ from the leading edge (see Fig. 1). Let the approaching supersonic stream be at a small angle of attack $\bar{\alpha}$. A shock wave will be formed below the lower surface of the airfoil and, depending on the relative values of $\bar{\alpha}$ and the upper surface initial angle, the upper surface may also have a shock wave. Assuming that the shock waves are attached to the apex, the two surfaces of the airfoil will act independently and we may consider the lower surface only. Choose a coordinate system $O\bar{x}\bar{y}$ fixed in space such that O coincides with the airfoil vertex, the $O\bar{x}$ axis coincides with a reference direction, and the $O\bar{y}$ axis points downward. Through careful analysis of the shock wave relations and body boundary condition, for both steady and unsteady flows, a characteristic perturbation parameter ϵ was defined[3] by

$$\epsilon = \frac{\gamma - 1}{\gamma + 1} + \frac{2}{(\gamma + 1)M_\infty^2} \tag{1}$$

and was used to derive the steady flow approximate equations under the assumption that $(\gamma - 1)M_\infty^2$ and $\bar{F}_o(\bar{x})/\sqrt{\epsilon}$, and $\bar{\alpha}/\sqrt{\epsilon}$ are fixed as $\epsilon \to 0$ (that is, as $\gamma \to 1$ and $M_\infty \to \infty$), where $\bar{F}_o(\bar{x})$ is the physical equation of the airfoil. Further, for the unsteady flow, the amplitude of pitching oscillation was assumed to be $\lambda\epsilon$, where $\lambda = \lambda_o e^{i\omega\bar{t}}$, $\lambda_o \ll 1$, is the amplitude of oscillation, ω the circular frequency, i the imaginary unity, and $\bar{t}$ the time variable. During its oscillatory motion, the airfoil surface can be approximated by

$$\bar{F}(\bar{x},\bar{t}) = \bar{F}_o(\bar{x}) + \lambda\epsilon(\bar{h} - \bar{x}) + O(\lambda_o^2\epsilon^2) \tag{2}$$

Consistent with the assumption that $\bar{F}_o(\bar{x})$ is $O(\sqrt{\epsilon})$ used in deriving the steady flow equations, the following nondimen-

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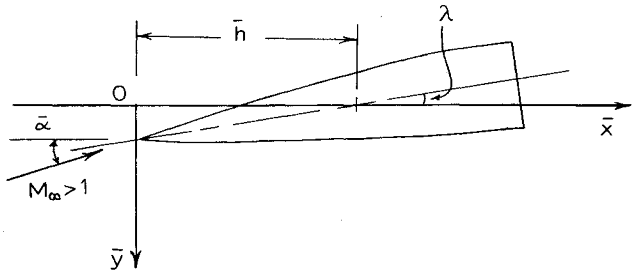

Fig. 1 Oscillating wing geometry and coordinate system.

sional variables and angle of attack were used in the derivation of the unsteady flow equations

$$x=\frac{\bar{x}}{l},\quad y=\frac{\bar{y}-\lambda\epsilon(\bar{h}-\bar{x})}{l\sqrt{\epsilon}},\quad t=\frac{\bar{t}U_\infty}{l},\quad \alpha=\frac{\bar{\alpha}}{\sqrt{\epsilon}} \tag{3}$$

where U_∞ is the freestream speed. The variables x, y, and α are assumed to be $O(1)$ in the flowfield as ϵ or $\lambda_o \to 0$. At the wing surface we have

$$y=\frac{\bar{F}_o(\bar{x})}{l\sqrt{\epsilon}}=G(x)$$

The asymptotic expansion as $\epsilon\to 0$ and $\lambda_o\to 0$ should thus be expressed in terms of x, y, and t. The following perturbation expansion was proposed[3] to describe the unsteady flow past pitching oscillating thin supersonic airfoils

$$\frac{\bar{u}(\bar{x},\bar{y},\bar{t})}{U_\infty}=[1+\epsilon u(x,y)+O(\epsilon^2)]+[\lambda\epsilon^{3/2}U_1(x,y) + O(\lambda\epsilon^{5/2})] \tag{4a}$$

$$\frac{\bar{v}(\bar{x},\bar{y},\bar{t})}{U_\infty}=[\sqrt{\epsilon}v(x,y)+O(\epsilon^{3/2})]+[\lambda\epsilon V_1(x,y)+O(\lambda\epsilon^2)] \tag{4b}$$

$$\frac{\bar{p}(\bar{x},\bar{y},\bar{t})-P_\infty}{\rho_\infty U_\infty^2}=[\epsilon p(x,y)+O(\epsilon^2)]+[\lambda\epsilon^{3/2}P_1(x,y) + O(\lambda\epsilon^{5/2})] \tag{4c}$$

$$\frac{\bar{\rho}(\bar{x},\bar{y},\bar{t})}{\rho_\infty}=[\rho(x,y)+O(\epsilon)]+[\lambda\sqrt{\epsilon}R_1(x,y)+O(\lambda\epsilon^{3/2})] \tag{4d}$$

$$\frac{\bar{S}(\bar{x},\bar{t})}{l}=[\sqrt{\epsilon}S(x)+O(\epsilon^{3/2})]+[\lambda\epsilon(h-x+S_1(x)) + O(\lambda\epsilon^2)] \tag{4e}$$

where $h=\bar{h}/l$, $k=\omega l/U_\infty$, k is the reduced frequency, and P_∞ and ρ_∞ are the freestream pressure and density, respectively. In Eqs. (4), the first square bracket gives the steady flow expansion and the second square bracket gives the unsteadiness perturbation. The steady flow functions u, v, p, ρ, and S are assumed to be $O(1)$ as $\epsilon\to 0$ and the unsteady flow functions U_1, V_1, P_1, R_1, and S_1 are assumed to be $O(1)$ as $\epsilon\to 0$ and $\lambda_o\to 0$. A basic assumption in Eqs. (4) is that they are uniformly valid in the flowfield of interest bounded by the lower surface of the airfoil and the shock wave. A nonuniformity might be expected for larger values of ϵ, specifically as $\epsilon\to 1$ (since $0<\epsilon<1$ for supersonic flow).

Substituting Eqs. (4) into the unsteady inviscid nonheat conducting governing equations, the body boundary condition, and the shock wave relations, and retaining the lowest order terms in ϵ and λ_o, a system of approximate equations for the steady flow was found.[3] In Ref. 2, a first integral to it was obtained as $\rho=1+(1+N)p$ (N is a constant defined later), thus reducing the steady flow to the following system 1 of equations:

System 1:

$$\tilde{p}_x+v\tilde{p}_y+v_y=0 \tag{5a}$$

$$n\tilde{p}_y+v_x+vv_y=0 \tag{5b}$$

$$v[x,G(x)]=G'(x) \tag{6}$$

$$v[x,S(x)]=S'(x)-\frac{1}{(1+N)[\alpha+S'(x)]} \tag{7a}$$

$$\tilde{p}[x,S(x)]=2\ln[\alpha+S'(x)] \tag{7b}$$

where

$$N=\left(\frac{\gamma-1}{2}\right)M_\infty^2$$

$$\tilde{p}(x,y)=\ln[n+p(x,y)]$$

Besides the steady flow equations, we get the following system 2 of equations and boundary conditions for the unsteady flow:

System 2:

$$\rho_y[V_1+1-ik(h-x)]+\rho V_{1y}+v_yR_1+ikR_1+R_{1x} + vR_{1y}=0 \tag{8a}$$

$$u_y[V_1+1-ik(h-x)]-\frac{p_x}{\rho^2}R_1+\frac{1}{\rho}p_y+\frac{1}{\rho}P_{1x}+ikU_1+U_{1x} + vU_{1y}=0 \tag{8b}$$

$$v_y[V_1+1-ik(h-x)]+vV_{1y}+V_{1x}+ikV_1-\frac{R_1}{\rho^2}p_y + \frac{1}{\rho}P_{1y}=0 \tag{8c}$$

$$p_y[V_1+1-ik(h-x)]+ikP_1+P_{1x}+vP_{1y}+v_yP_1 -\frac{1}{\rho}\left[p+\frac{1}{1+N}\right]\{v_yR_1+\rho_y[V_1+1-ik(h-x)]+ikR_1 + R_{1x}+vR_{1y}\}=0 \tag{8d}$$

at $y=G(x)$,

$$V_1=ik(h-x)-1 \tag{9}$$

at $y=S(x)$,

$$P_1=2(\alpha+S')\phi(x)-S_1p_y(x,S) \tag{10a}$$

$$R_1=2(1+N)(\alpha+S')\phi(x)-S_1\rho_y(x,S) \tag{10b}$$

$$V_1=\phi(x)\left[1+\frac{1}{(1+N)(\alpha+S')^2}\right]-S_1v_y(x,S) \tag{10c}$$

$$U_1=-S_1u_y(x,S)+(\alpha+S')(1-S_1')-S'\phi(x) -\frac{(1-S_1')}{(1+N)(\alpha+S')}-\frac{S'\phi(x)}{(1+N)(\alpha+S')^2} \tag{10d}$$

where

$$\phi(x)=S_1'(x)-1+ik[h-x+S_1(x)]$$

[and the positive sign of the term v_yP_1 in Eq. (8d) corrects the negative sign given in Ref. (3)]. System 1 is nonlinear and gives the steady flow past pointed-nose thin airfoils at small angles of attack. It has been studied in detail in Refs. 1, 2, 22, and 23. System 2 is linear and gives the unsteady flow past (small amplitude) pitching oscillating thin airfoils at small incidence provided that the shock wave is attached. Notice that U_1 is decoupled from the equations of system 2, therefore Eqs. (8b) and (10d) will be disregarded in what follows. It was noted in Ref. 1 that, although the limiting process that led to system 1 is rather unusual, it is sound and has, among other verifications, given a correct Newtonian limit when a further expansion based on the limit $N\to\infty$ was applied to it. In fact, in this case, the equations are reduced to Cole's[24] zeroth-order Newtonian theory. Other results of system 1 at moderate and hypersonic speeds have shown very good agreement with other exact and approximate theories. System 2 was used[3] before to derive a zeroth-order unsteady Newtonian theory by taking the additional limit $N\to\infty$, whereas in Ref. 4, the limit $N\to 0$ was applied to it to obtain approximate equations applicable at moderate supersonic Mach numbers.

It is the purpose of this paper to study system 2 for the full range of N in order to cover hypersonic speeds as well, thus including the results of Refs. 3 and 4 as special cases. First, we make the transformation

$$n = \frac{1}{1+N}, \qquad \bar{R}(x,y) = nR_1(x,y)$$

Equations (8d), (10a), and (10b) become

$$\left(ik + \frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + \frac{\partial v}{\partial y}\right)(P_1 - \hat{R}) = 0 \tag{11a}$$

$$P_1[x,S(x)] = 2[\alpha + S'(x)]\phi(x) - S_1(x)p_y[x,S(x)] \tag{11b}$$

$$\hat{R}[x,S(x)] = 2[\alpha + S'(x)]\phi(x) - nS_1(x)\rho_y[x,S(x)] \tag{11c}$$

Equations (11a–c) imply that the functions P_1 and $\hat{R}$ are equal all over the flowfield. By this first integral, we get rid of the function $\hat{R}$ and reduce system 2 to the following simpler form:

Unsteady flow:

$$\{\exp(\tilde{p})[V_1 + 1 + ik(x-h)]\}_y + (v_y + ik)P_1 + P_{1x} + vP_{1y} = 0 \tag{12a}$$

$$v_y[V_1 + 1 + ik(x-h)] + vV_{1y} + V_{1x} + ikV_1 + n[P_1\exp(-\tilde{p})]_y = 0 \tag{12b}$$

$$V_1[x,G(x)] = ik(h-x) - 1 \tag{13}$$

$$P_1[x,S(x)] = 2(\alpha + S')\phi(x) - S_1\tilde{p}_y(x,S)\exp[\tilde{p}(x,S)] \tag{14a}$$

$$V_1[x,S(x)] = \phi(x)\left[1 + \frac{n}{(\alpha+S')^2}\right] - S_1v_y(x,S) \tag{14b}$$

III. Perturbation Scheme

In this section, we seek a solution to Eqs. (12–14) as a small perturbation from some basic wedge flow for airfoils that are slightly curved. First, for such airfoils, a solution to the steady flow equations was assumed[23] as

$$v(x,y) = v_o + \tau x v_1(\eta) + \tau^2x^2v_2(\eta) + \ldots \tag{15a}$$

$$\tilde{p}(x,y) = p_o + \tau x p_1(\eta) + \tau^2x^2p_2(\eta) + \ldots \tag{15b}$$

$$F(x) = Bx + \tau x^2 + b\tau^2x^3 + \ldots \tag{15c}$$

$$S(x) = Ax + A_1\tau x^2 + A_2\tau^2x^3 + \ldots \tag{15d}$$

where η is a similarity parameter defined as $\eta = y/x$, and p_o, v_o, B, and A are constants giving the basic wedge flow, whereas v_1, v_2, p_1, and p_2 are functions of the single variable η, which are assumed to be $O(1)$ as $\tau\to 0$. The body constants B, τ, and b are assumed to be known, whereas the shock wave constants A, A_1, A_2 are to be found as part of the solution. The results of this approach have shown very good agreement with other exact and approximate methods (see Ref. 23), especially at hypersonic Mach numbers. Here, we restrict ourselves to terms $O(\tau)$ only and neglect second-order terms in Eqs. (15). Following the same lines, we assume the following perturbation expansion to the unsteady flow equations (12–14) as $\tau\to 0$ and $k\to 0$

$$P_1(x,y) = [r_o + \tau r_2(x,y) + \ldots] + ik[r_1(x,y) + \tau r_3(x,y) + \ldots] + O(k^2) \tag{16a}$$

$$V_1(x,y) = [g_o + \tau g_2(x,y) + \ldots] + ik[g_1(x,y) + \tau g_3(x,y) + \ldots] + O(k^2) \tag{16b}$$

$$S_1(x) = [A_ox + \tau s_2(x)] + ik[s_1(x) + \tau s_3(x) + \ldots] + O(k^2) \tag{16c}$$

where r_o, g_o, and A_o are constants corresponding to the unsteady basic wedge flow, and the functions r_1, g_1, and s_1 give the wedge perturbation and are assumed to be $O(1)$ as $k\to 0$, whereas the functions r_2, r_3, g_2, g_3, s_2, and s_3 give the effect of surface curvature and are also assumed to be $O(1)$ as τ and $k\to 0$.

Substituting Eqs. (15) into Eqs. (5–7) and equating like powers of τ, a set of equations for steady flow has been found,[23] which will not be written here. We only write the needed results as follows.

$$v_o = B, \qquad p_o = 2\ell n(\alpha + A)$$

$$A = \tfrac{1}{2}[B - \alpha + \sqrt{(B+\alpha)^2 + 4n}]$$

$$v_1(\eta) = \alpha_1\eta + \alpha_o, \qquad p_1(\eta) = c_1\eta + c_o$$

$$c_o = \frac{2B}{n} - \alpha_1, \qquad c_1 = \frac{-2}{n}, \qquad A_1 = \frac{n - (A-B)^2}{n + 3(A-B)^2}$$

$$\alpha_o = 2 - B\alpha_1, \qquad \alpha_1 = \frac{2}{n}(B-A)(1+2A_1)$$

Similarly, substituting Eqs. (16) into Eqs. (12–14) and equating like powers of τ, k, and τk, we first get

$$g_o = -1 = (A_o - 1)\left[1 + \frac{n}{(\alpha+A)^2}\right] \tag{17a}$$

$$r_o = 2(\alpha + A)(A_o - 1) \tag{17b}$$

The solution of Eqs. (17) is given by

$$A_o = \frac{n}{n + (\alpha+A)^2}$$

$$r_o = \frac{-2(\alpha+A)^3}{n + (\alpha+A)^2}$$

We then get the following A, B, and C sets of equations:

Set A:

$$(\alpha + A)^2 g_{1y} + r_{1x} + B r_{1y} = -r_o \tag{18a}$$

$$B g_{1y} + g_{1x} + \frac{n}{(\alpha + A)^2} r_{1y} = -g_o \tag{18b}$$

$$g_1(x, Bx) = h - x \tag{18c}$$

$$r_1(x, Ax) = 2(\alpha + A)(s_1' + h - x + A_o x) \tag{18d}$$

$$g_1(x, Ax) = (s_1' + h - x + A_o x)\left[1 + \frac{n}{(\alpha + A)^2}\right] \tag{18e}$$

Set B:

$$(\alpha + A)^2 g_{2y} + r_{2x} + B r_{2y} = -\alpha_1 r_o \tag{19a}$$

$$B g_{2y} + g_{2x} + \frac{n}{(\alpha + A)^2} r_{2y} = \frac{n c_1 r_o}{(\alpha + A)^2} \tag{19b}$$

$$g_2(x, Bx) = 0 \tag{19c}$$

$$r_2(x, Ax) = 2(\alpha + A) s_2' + 4A_1(A_o - 1)x$$
$$- c_1 A_o (\alpha + A)^2 x \tag{19d}$$

$$g_2(x, Ax) = \frac{-4nA_1(A_o - 1)x}{(\alpha + A)^3} - A_o \alpha_1 x$$
$$+ s_2'\left[1 + \frac{n}{(\alpha + A)^2}\right] \tag{19e}$$

Set C:

$$(\alpha + A)^2 g_{3y} + r_{3x} + B r_{3y} = -(\alpha + A)^2 [c_1(g_1 - h + x)$$
$$+ g_{1y}(c_1 y + c_o x)] - \alpha_1 r_1 - r_2 - (\alpha_1 y + \alpha_o x) r_{1y} \tag{20a}$$

$$B g_{3y} + g_{3x} + \frac{n}{(\alpha + A)^2} r_{3y} = \alpha_1 (h - x - g_1) - (\alpha_1 y + \alpha_o x) g_{1y}$$
$$- g_2 + \frac{n}{(\alpha + A)^2}[c_1 r_1 + (c_1 y + c_o x) r_{1y}] \tag{20b}$$

$$g_3(x, Bx) = -x^2 g_{1y}(x, Bx) \tag{20c}$$

$$r_3(x, Ax) = 2(\alpha + A)[s_3'(x) + s_2(x)] - A_1 x^2 r_{1y}(x, Ax)$$
$$+ 4A_1 x [s_1'(x) + h - x + A_o x] - c_1 (\alpha + A)^2 s_1(x) \tag{20d}$$

$$g_3(x, Ax) = -A_1 x^2 g_{1y}(x, Ax) - \alpha_1 s_1(x)$$
$$+ [s_3'(x) + s_2(x)]\left[1 + \frac{n}{(\alpha + A)^2}\right]$$
$$- \frac{4nA_1 x}{(\alpha + A)^3}[s_1'(x) + h - x + A_o x] \tag{20e}$$

IV. Solution for Curved Airfoils

Sets A, B, and C are linear with constant coefficients and their boundary conditions are quite simple, therefore, their solutions are straightforward. First, the shock waves $s_1(x)$, $s_2(x)$, and $s_3(x)$ could be eliminated from the formulation. Starting with set A, and upon eliminating $s_1'(x)$ from Eqs. (18d) and (18e), we get

$$r_1(x, Ax) + r_o g_1(x, Ax) = 0 \tag{21}$$

It can easily be shown that the solution of Eqs. (18a–c) and (21) exists in the following form

$$r_1(x, y) = \bar{r}_1 + \hat{r}_1 x + \tilde{r}_1 y \tag{22a}$$

$$g_1(x, y) = \bar{g}_1 + \hat{g}_1 x + \tilde{g}_1 y \tag{22b}$$

where $\bar{r}_1$, $\hat{r}_1$, $\tilde{r}_1$, $\bar{g}_1$, $\hat{g}_1$, and $\tilde{g}_1$ are constants. Substituting Eqs. (22) into Eqs. (18a–c) and Eq. (21) and equating like powers of x and y, we get

$$\bar{g}_1 = h, \qquad \bar{r}_1 = -r_o h, \qquad \tilde{r}_1 = \frac{2}{n}(\alpha + A)^2$$

$$\tilde{g}_1 = \frac{-2nr_o + 2(A - B)(\alpha + A)^2}{n[(\alpha + A)^2 - r_o(A - B)]}$$

$$\hat{g}_1 = -1 - B\tilde{g}_1, \qquad \hat{r}_1 = r_o - \frac{2}{n}A(\alpha + A)^2 - r_o \tilde{g}_1 (A - B)$$

Set B can be solved the same way. Thus, upon eliminating $s_2'(x)$ from Eqs. (19d) and (19e), we get

$$r_2(x, Ax) + r_o g_2(x, Ax) = \beta x \tag{23}$$

where

$$\beta = -r_o\left[A_o \alpha_1 + \frac{4nA_1(A_o - 1)}{(\alpha + A)^3}\right] + 4A_1(A_o - 1)$$
$$- c_1 A_o (\alpha + A)^2$$

and the solution of Eqs. (19a–c) and Eq. (23) can be assumed as

$$g_2(x, y) = \bar{g}_2 + \hat{g}_2 x + \tilde{g}_2 y \tag{24a}$$

$$r_2(x, y) = \bar{r}_2 + \hat{r}_2 x + \tilde{r}_2 y \tag{24b}$$

where $\bar{g}_2$, $\hat{g}_2$, $\tilde{g}_2$, $\bar{r}_2$, $\hat{r}_2$, and $\tilde{r}_2$ are constants. Following a similar procedure, we get

$$\bar{g}_2 = \bar{r}_2 = 0, \qquad \tilde{r}_2 = c_1 r_o$$

$$\hat{g}_2 = -B\tilde{g}_2, \qquad \hat{r}_2 = -r_o(\alpha_1 + Bc_1) - (\alpha + A)^2 \tilde{g}_2$$

$$\tilde{g}_2 = \frac{r_o[c_1(A - B) - \alpha_1] - \beta}{(\alpha + A)^2 - r_o(A - B)}$$

Finally, to solve set C, we first substitute the previous results into Eqs. (20) and eliminate $s_3'(x)$ to get

$$(\alpha + A)^2 g_{3y} + r_{3x} + B r_{3y} = l_o + l_1 x + l_2 y \tag{25a}$$

$$B g_{3y} + g_{3x} + \frac{n}{(\alpha + A)^2} r_{3y} = \bar{l}_o + \bar{l}_1 x + \bar{l}_2 y \tag{25b}$$

$$g_3(x, Bx) = -\tilde{g}_1 x^2 \tag{25c}$$

$$r_3(x, Ax) + r_o g_3(x, Ax) = (\hat{l}_o + r_o \bar{l}_o)x + (\hat{l}_1 + r_o \bar{l}_1)x^2 \tag{25d}$$

where

$$\hat{l}_o = \frac{2A_1\bar{r}_1}{(\alpha + A)} - c_1(\alpha + A)^2 l_o^*$$

$$\tilde{l}_o = -\alpha_1 l_o^* - \frac{2nA_1\bar{r}_1}{(\alpha + A)^4}$$

$$\hat{l}_1 = -A_1\tilde{r}_1 + \frac{2A_1}{(\alpha + A)}(\hat{r}_1 + \tilde{r}_1 A) - c_1(\alpha + A)^2 l_1^*$$

$$\tilde{l}_1 = -A_1\tilde{g}_1 - \alpha_1 l_1^* - \frac{2nA_1}{(\alpha + A)^4}(\hat{r}_1 + \tilde{r}_1 A)$$

$$l_o^* = \frac{\bar{r}_1}{2(\alpha + A)} - h$$

$$l_1^* = \frac{1}{2}\left[\frac{(\alpha + A)^2}{n + (\alpha + A)^2} + \frac{\hat{r}_1 + \tilde{r}_1 A}{2(\alpha + A)}\right]$$

$$l_o = \alpha_1 r_o h$$

$$l_1 = (\alpha + A)^2 c_1 B\tilde{g}_1 - \alpha_1\hat{r}_1 - \hat{r}_2 - \tilde{r}_1\alpha_o - c_o\tilde{g}_1(\alpha + A)^2$$

$$l_2 = -2c_1\tilde{g}_1(\alpha + A)^2 - 2\alpha_1\tilde{r}_1 - c_1 r_o$$

$$\bar{l}_o = \frac{-nc_1 r_o h}{(\alpha + A)^2}$$

$$\bar{l}_1 = \alpha_1 B\tilde{g}_1 + B\tilde{g}_2 + \frac{nc_1\hat{r}_1}{(\alpha + A)^2} + \frac{n\tilde{r}_1 c_o}{(\alpha + A)^2} - \alpha_o\tilde{g}_1$$

$$\bar{l}_2 = -2\alpha_1\tilde{g}_1 - \tilde{g}_2 + \frac{2nc_1\tilde{r}_1}{(\alpha + A)^2}$$

The solutions of Eqs. (25) exist in the form

$$g_3(x, y) = \tilde{g}_3 x^2 + \hat{g}_3 xy + \bar{g}_3 y^2 + g_3^* x + \bar{g}_3^* y \tag{26a}$$

$$r_3(x, y) = \tilde{\rho}_3 x^2 + \hat{\rho}_3 xy + \bar{\rho}_3 y^2 + \rho_3^* x + \bar{\rho}_3^* y \tag{26b}$$

where g_3 and ρ_3 with superscripts $\tilde{\cdot}$, $\hat{\cdot}$, $\bar{\cdot}$, *, and $\bar{\cdot}^*$ are constants. Following a similar procedure to that used earlier, we first get

$$\bar{\rho}_3^* = \frac{\bar{l}_o}{n}(\alpha + A)^2$$

$$\rho_3^* = \frac{[A(\alpha + A)^2 - B(A - B)]\bar{\rho}_3^*}{[A - B - (\alpha + A)^2]} + \frac{l_o(A - B) - (\hat{l}_o + r_o\tilde{l}_o)(\alpha + A)^2}{[A - B - (\alpha + A)^2]}$$

$$\bar{g}_3^* = \frac{1}{(A - B)}(\hat{l}_o + r_o\tilde{l}_o - A\tilde{r}_3 - \hat{r}_3)$$

$$g_3^* = -B\bar{g}_3^*$$

then the constants $\bar{g}_3$, $\hat{g}_3$, $\tilde{g}_3$, $\bar{\rho}_3$, $\hat{\rho}_3$, and $\tilde{\rho}_3$ are to be determined by the following linear equation

$$\begin{pmatrix} 0 & q_1 & 0 & B & 2 & 0 \\ 2q_1 & 0 & 0 & 1 & 0 & 2B \\ 0 & B & 2 & q_2 & 0 & 0 \\ 2B & 1 & 0 & 0 & 0 & 2q_2 \\ q_3 & q_4 & r_o & A & 1 & A^2 \\ -B^2 & -B & -1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \bar{g}_3 \\ \hat{g}_3 \\ \tilde{g}_3 \\ \hat{\rho}_3 \\ \bar{\rho}_3 \\ \tilde{\rho}_3 \end{pmatrix} = \begin{pmatrix} l_1 \\ l_2 \\ \bar{l}_1 \\ \bar{l}_2 \\ l_3 \\ \tilde{g}_1 \end{pmatrix} \tag{27}$$

where

$$q_1 = (\alpha + A)^2, \qquad q_2 = n/(\alpha + A)^2$$

$$q_3 = r_o A^2, \qquad q_4 = r_o A$$

$$l_3 = \hat{l}_1 + r_o\tilde{l}_1$$

Equation (27) can easily be solved using a digital computer.

V. Stability Derivatives of Nonsymmetric Airfoils

The unsteady lower surface pressure coefficient will be denoted by C_{pl}, where l refers to the lower surface. As usual, C_{pl} is defined by

$$C_{pl} = \frac{\bar{p}[\bar{x}, \bar{F}_l(\bar{x}, \bar{t}), \bar{t}] - P_\infty}{\frac{1}{2}\rho_\infty U_\infty^2}$$

$$= 2\epsilon p_l[x, G_l(x)] + 2\lambda_l\epsilon^{3/2}P_{1l}[x, G_l(x)]$$

$$\equiv C_{p_{ol}} + 2\lambda_l\epsilon^{3/2}P_{1l}[x, G_l(x)]$$

where p_l, P_{1l} refer to values of p and P_1 at the lower surface $\bar{F}_l(\bar{x}, \bar{t})$ or $G_l(x)$, λ_l is the lower surface amplitude of oscillation, and $C_{p_{ol}}$ is the lower surface steady pressure coefficient. Similarly, the upper surface unsteady pressure coefficient C_{pu} can be defined. Now,

$$(C_{pl} - C_{pu}) - (C_{p_{ol}} - C_{p_{ou}}) = 2\lambda\epsilon^{3/2}\{P_{1l}[x, G_l(x)] + P_{lu}[x, G_u(x)]\}$$

The coefficient of pitching moment C_m is defined by

$$C_m = \frac{M_h}{\frac{1}{2}\rho_\infty U_\infty^2 l^2}$$

where M_h is the pitching moment per unit length. Using previous results for the surface pressure coefficient, we get

$$C_m = \int_0^1 [(C_{pl} - C_{pu}) - (C_{p_{ol}} - C_{p_{ou}})](x - h)\,\mathrm{d}x$$

$$= 2\lambda\epsilon^{3/2}\int_0^1 (x - h)\{P_{1l}[x, G_l(x)] + P_{lu}[x, G_u(x)]\}\,\mathrm{d}x$$

Since the unsteady perturbation pressure $P_1(x, y)$ has been found, there is no difficulty in finding C_m in closed form. Now,

$$C_m = \lambda\epsilon[C_{m\theta} + ikC_{m\dot{\theta}}]$$

where $-C_{m\theta}$ and $-C_{m\dot{\theta}}$ are the stiffness and damping derivatives, respectively.

After some calculations we get

$$-C_{m_\theta} = \sqrt{\epsilon}(2h - 1)\Sigma(r_o + \tau\bar{r}_2) + \sqrt{\epsilon}\left(h - \frac{2}{3}\right)\Sigma\tau(\hat{r}_2 + B\tilde{r}_2) \tag{28a}$$

$$-C_{m_{\dot{\theta}}} = \sqrt{\epsilon}(2h - 1)\Sigma\bar{r}_1 + \sqrt{\epsilon}\left(h - \frac{2}{3}\right)\Sigma(\hat{r}_1 + B\tilde{r}_1 + \tau\rho_3^* + \tau B\bar{\rho}_3^*) + \sqrt{\epsilon}\left(\frac{2}{3}h - \frac{1}{2}\right)\Sigma\tau(\tilde{\rho}_3 + B\hat{\rho}_3 + B^2\bar{\rho}_3) \tag{28b}$$

In Eqs. (28), the summation is taken over the upper and lower surfaces of the airfoil. Equations (28a) and (28b) give simple closed-form formulas for the stiffness and damping derivatives of thin pointed-nose airfoils making small-amplitude

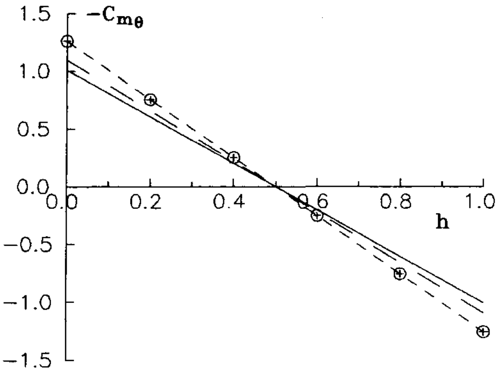

Fig. 2 Comparison for $-C_{m_\theta}$ for a symmetric wedge ($\delta = 6.85$ deg, $\bar{\alpha} = 0$; $\gamma = 1.4$; $M_\infty = 2.47$): —— Eq. (30a); - - - Hui[10]; ○ Pugh and Woodgate (experiment); —— piston theory.

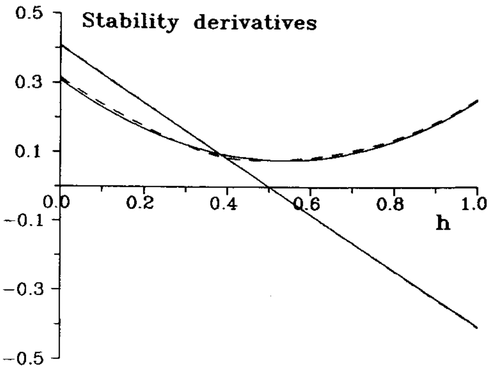

Fig. 4 Comparison for the stability derivatives of a wedge ($\delta = 4.75$ deg; $\bar{\alpha} = 0$; $\gamma = 1.405$; $M_\infty = 9.7$): —— Eqs. (30); - - - Orlik-Rückemann.[9]

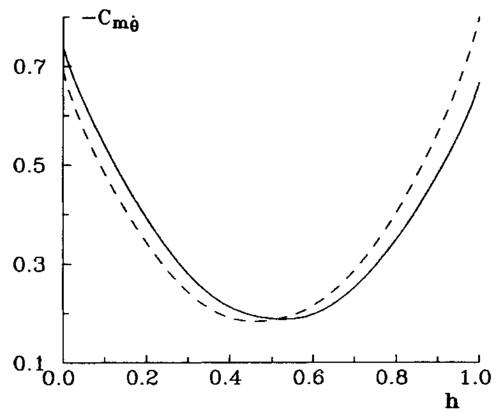

Fig. 3 Comparison for the damping derivative of a symmetric wedge ($\delta = 10$ deg; $\bar{\alpha} = 0$; $\gamma = 1.4$; $M_\infty = 3$): —— Eq. (30b); - - - Hui.[10]

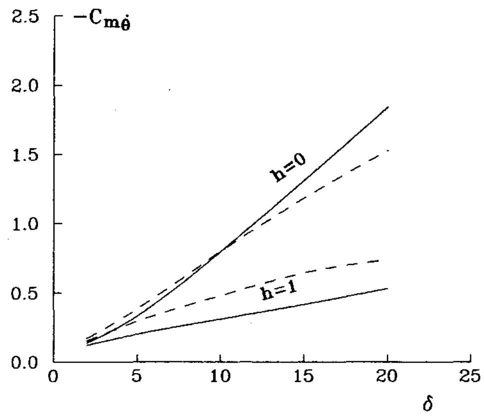

Fig. 5 Comparison for the damping derivative ($\bar{\alpha} = 0$, $\gamma = 1.4$, $M_\infty = 17$): —— Eq. (30b); - - - Hui.[10]

pitching oscillations at small angles of attack. The two surfaces of the airfoil are curved and their equations can be different. Notice that the constants in Eqs. (28a) and (28b) (except h and ϵ) will have different values for the two surfaces because of the angle of attack and different shapes of the upper and lower surfaces. At the range of (small) angles of attack considered here, the effect of the two surfaces of the airfoil on the stability derivatives should be taken into consideration and it is very likely that they will have different shapes. The special case of a symmetric curved airfoil at zero incidence will have $F_u = F_l$ and $\bar{\alpha} = 0$; thus, the stability derivatives in this case will be

$$-C_{m_\theta} = 2\sqrt{\epsilon}(2h - 1)(r_o + \tau\bar{r}_2) + 2\sqrt{\epsilon}\tau\left(h - \frac{2}{3}\right)(\hat{r}_2 + B\bar{r}_2) \tag{29a}$$

$$-C_{m_{\dot{\theta}}} = 2\sqrt{\epsilon}(2h - 1)\bar{r}_1 + 2\sqrt{\epsilon}\left(h - \frac{2}{3}\right)(\hat{r}_1 + B\bar{r}_1 + \tau\rho_3^* + \tau B\bar{\rho}_3^*) + \sqrt{\epsilon}\tau\left(\frac{4}{3}h - 1\right)(\tilde{\rho}_3 + B\hat{\rho}_3 + B^2\bar{\rho}_3) \tag{29b}$$

The special case of a symmetric wedge at zero incidence can be recovered from Eqs. (29) by setting $\tau = 0$; thus, for such wedges, we have

$$-C_{m_\theta} = \left(\frac{1}{2} - h\right)\frac{8\sqrt{\epsilon}A^3}{n + A^2} \tag{30a}$$

$$-C_{m_{\dot{\theta}}} = \frac{8\sqrt{\epsilon}A^3}{n + A^2}\left[h^2 - \frac{h}{2r_o}(r_o + \hat{r}_1 + B\bar{r}_1) + \frac{1}{3r_o}(\hat{r}_1 + B\bar{r}_1)\right] \tag{30b}$$

VI. Results and Comparisons

In this section, we present results for the stiffness and damping derivatives of symmetric and nonsymmetric curved airfoils at small angles of attack. We also compare the theory with the available results for symmetric wedges at zero incidence.

Figure 2 compares Eq. (30a) for the stiffness derivative with Hui's[10] theory, the piston theory, and the experiment of Pugh and Woodgate.[25] (In Figs. 2–5, δ refers to the wedge semivertex angle.) The agreement with the piston theory is shown to be good, but with Hui's theory and experiment the agreement is only modest. This could possibly be due to M_∞ being too low for the present theory to hold out. Figure 3 compares Eq. (30b) for the damping derivative with Hui's[10] theory. Again, the agreement is shown to be modest.

In Fig. 4, Eqs. (30a) and (30b) are compared with Orlik-Ruckemann's[9] theory (complete result including wave reflec-

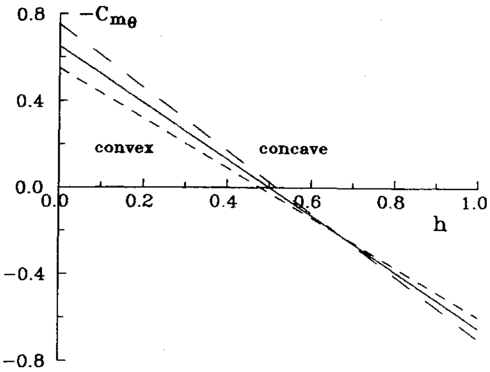

Fig. 6 Effect of surface concave and convex curvature on $-C_{m_\theta}$ ($F_u = 0$; $F_l = 0.1x + \tau x^2$; $\bar{\alpha} = 10$ deg; $\gamma = 1.4$; $M_\infty = 5$): —— $\tau = 0$; — — $\tau = 0.03$; - - $\tau = -0.03$.

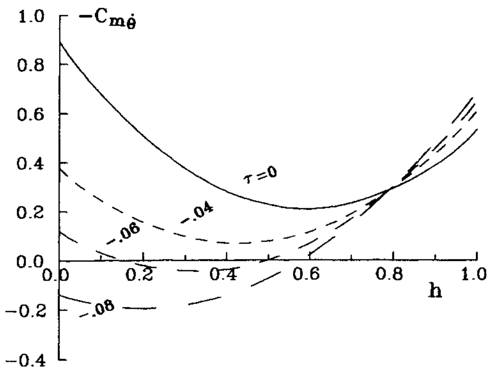

Fig. 8 Effect of surface convex curvature on the damping derivative: $F_u = F_l = 0.2x + \tau x^2$; $\bar{\alpha} = 10$ deg; $\gamma = 1.4$; $M_\infty = 5$.

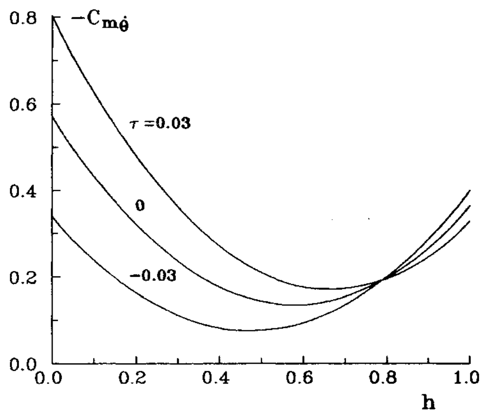

Fig. 7 Effect of surface concave and convex curvature on the damping derivative: $F_u = 0$; $F_l = 0.1x + \tau x^2$; $\bar{\alpha} = 10$ deg; $\gamma = 1.4$; $M_\infty = 5$.

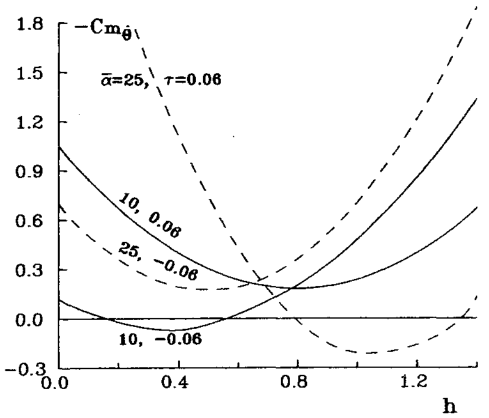

Fig. 9 Schematic layout of present theory (—— small $\bar{\alpha}$) and that of Ref. 21 (- - - moderate or large $\bar{\alpha}$): $F_u = F_l = 0.15x + \tau x^2$; $\gamma = 1.1$; $M_\infty = 6$.

tions). The agreement is seen to be almost perfect because the values of the wedge angle (9.5 deg) and M_∞ (9.7) are quite appropriate for the application of the present theory. Figure 5 compares Eq. (30b) with Hui's[10] theory for various wedges and for $h = 0$ and 1. Only modest agreement is obtained, which improves for small wedge angles. Since M_∞ is high in this case, greater agreement was expected at least for small values of δ. The reason for the modest agreement obtained is not clear.

Figures 6 and 7 show the effect of surface concave and convex curvature on the damping derivatives of a nonsymmetric airfoil. Figure 6 shows that the aerodynamic center is not at $h = 0.5$ (as for wedges) but moves a little backward for concave airfoils and a little forward for convex airfoils. Similar results were obtained in Ref. 21 for airfoils oscillating at moderate or large angles of attack in Newtonian flow, but in Ref. 4 (where the main steady and unsteady theories used here were further approximated for $n \to 0$ for moderate supersonic speeds at zero incidence), the aerodynamic center was found to be at $h = 0.5$ for concave, convex, and wedge airfoils. Figure 7 shows that convex curvature makes the airfoil less stable dynamically for pivot positions near the leading edge. Similar results were obtained in Refs. 4 and 21 (in Ref. 4, this occurs for all pivot positions).

Figure 8 shows that not only convex curvature reduces the stability derivative for pivot positions around the leading edge but it also tends to make the pitching oscillations dynamically unstable for certain pivot positions near the leading edge. Similar results have been obtained in Ref. 4 (see Ref. 4, Fig. 5); whereas in Ref. 21, it was found that, although convex curvature decreases the damping derivative for certain forward pivot positions, it is the concave curvature that tends to make the pitching oscillations, with pivot positions near the trailing edge, dynamically unstable (Ref. 21, Fig. 5). Figure 9 is a schematic diagram illustrating these conclusions.

Figures 10–12 show, respectively, the effect of the angle of attack on the damping derivative of 1) a symmetric convex airfoil, 2) a nonsymmetric convex airfoil, and 3) a symmetric concave airfoil. The three figures show that the angle of attack tends to increase the dynamic stability of concave or convex airfoils for pivot positions given by $h < \sim 0.7$. It should be noted that the angle-of-attack effect is the same as the effect of increasing the airfoil lower surface thickness and decreasing its upper surface thickness, both with the same $\bar{\alpha}$, because of the shock wave attachment assumption used. However, it is very convenient to have the angle of attack explicitly in the formulation.

Finally, in the absence of other theoretical or experimental results (one would expect the existence of few experimental results in research laboratories) to compare with the present theory in the case of curved surfaces, it is necessary to state clearly and emphasize the assumptions on which the theory is based. First, the limit $\gamma \to 1$ and M_∞ was applied to the steady governing equations in such a way that $(\gamma - 1)M_\infty^2$, $\bar{F}_o(\bar{x})/\sqrt{\epsilon}$, and $\bar{\alpha}/\sqrt{\epsilon}$ remain fixed, where ϵ is the basic perturbation parameter defined by Eq. (1). Second, the unsteady theory is based on the limit $\lambda_o \to 0$ with the assumption that the amplitude of oscillation is equal to $\lambda_o \epsilon$. Finally, the limit $\tau \to 0$ and $k \to 0$ is taken in this paper. With orders of magnitude as given earlier, the correct range of use of the theory should have $\gamma \to 1$ and $M_\infty \to \infty$ and all of the parameters $\bar{\alpha}$, airfoil thickness, slope, curvature, λ_o, and k should be small.

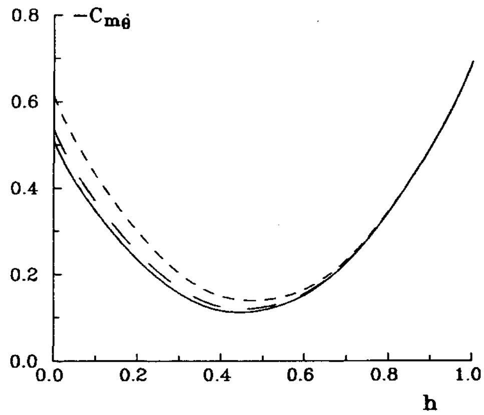

Fig. 10 Variation of $-C_{m_{\dot\theta}}$ for a symmetric convex airfoil [$F_u = F_l = \ell n(1 + 0.25x)$; $\gamma = 1.1$; $M_\infty = 5$]: —— $\bar\alpha = 0$; — — $\bar\alpha = 5$; - - - $\bar\alpha = 10$ deg.

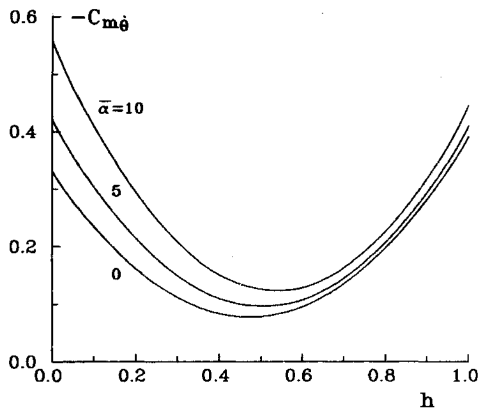

Fig. 11 Variation of $-C_{m_{\dot\theta}}$ for a nonsymmetric convex airfoil $F_u = 0.05x - 0.01x^2$ $F_l = 0.2x - 0.02x^2$; $\gamma = 1.105$; $M_\infty = 7$.

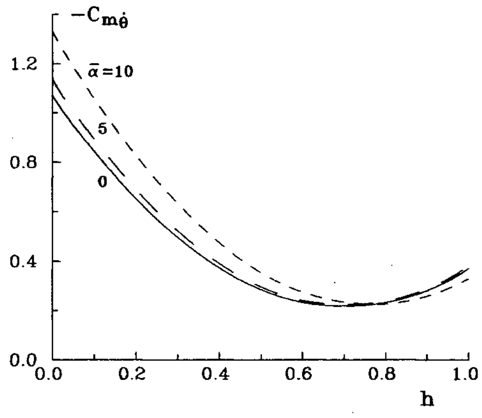

Fig. 12 Variation of $-C_{m_{\dot\theta}}$ for a nonsymmetric convex airfoil $F_u = F_l = 0.1763x + 0.03x^2$; $\gamma = 1.105$; $M_\infty = 7$.

VII. Conclusions

Simple closed-form formulas for the stability derivatives of small-amplitude pitching oscillations of thin sharp-edged nonsymmetric airfoils at small angles of attack are given in this paper. The analysis relies on recent hypersonic steady and unsteady theories based on the limit $\gamma \to 1$ and $M_\infty \to \infty$ such that $(\gamma - 1)M_\infty^2$, $\bar F_o(\bar x)/\sqrt{\epsilon}$, and $\bar\alpha/\sqrt{\epsilon}$, remain fixed, and λ_o is small. Further, a perturbation expansion is given here based on the limit $\tau \to 0$ and $k \to 0$ and, therefore, is applicable for airfoils with small surface curvature oscillating at small reduced frequency. A first integral for the unsteady governing equations is obtained whereby the density is eliminated from the formulation and the equations are reduced to linear partial differential equations with constant coefficients for which first- or second-degree polynomial solutions are assumed. Results for symmetric wedges at zero incidence are compared wtih other available theoretical and experimental results and the agreement is found to be generally good.

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